Example: Ampere's Law of Force

Let's again consider Ampere's Law of Force in the following form:

$$d\mathbf{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi} \frac{\left(\overline{d\ell_1} \cdot \hat{\mathbf{a}}_{21}\right)\overline{d\ell_2} - \left(\overline{d\ell_1} \cdot \overline{d\ell_2}\right)\hat{\mathbf{a}}_{21}}{R^2}$$

$$= \left(\frac{\mu_0 I_1 I_2}{4\pi R^2}\right)\left(\overline{d\ell_1} \cdot \hat{\mathbf{a}}_{21}\right)\overline{d\ell_2} \; - \; \left(\frac{\mu_0 I_1 I_2}{4\pi R^2}\right)\left(\overline{d\ell_1} \cdot \overline{d\ell_2}\right)\hat{\mathbf{a}}_{21}$$

It is apparent that we can consider the force on **filament 1** to consist of **two** forces, i.e.:

$$d\mathbf{F}_1 = d\mathbf{F}_1^a + d\mathbf{F}_1^b$$

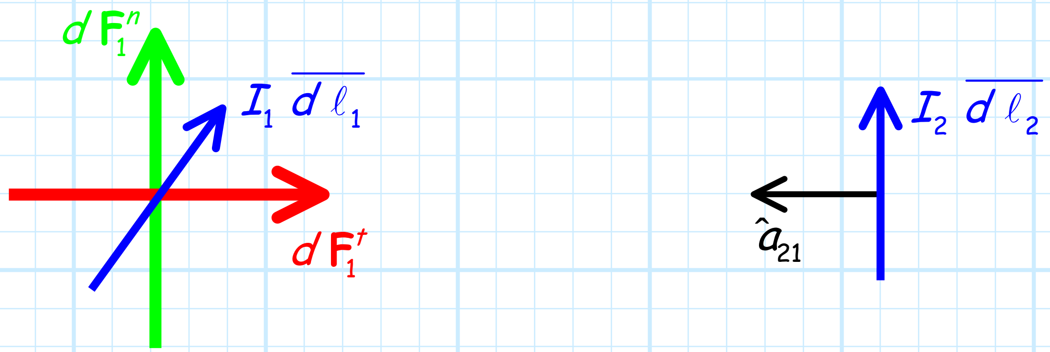
where

$$d\mathbf{F}_1^a = \left(\frac{\mu_0 I_1 I_2}{4\pi R^2}\right)\left(\overline{d\ell_1} \cdot \hat{\mathbf{a}}_{21}\right) \overline{d\ell_2}$$

and

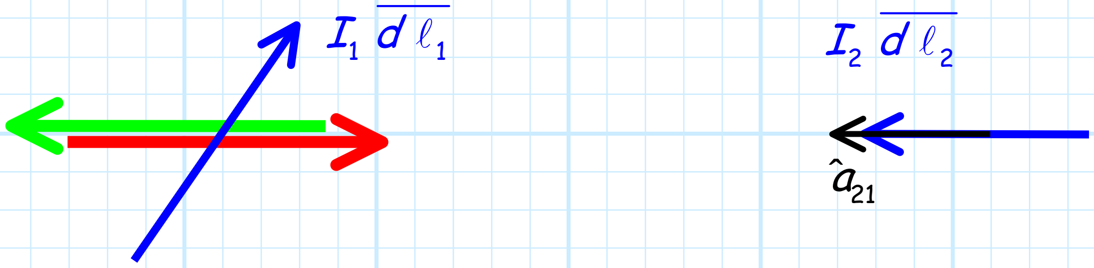
$$d\mathbf{F}_1^b = -\left(\frac{\mu_0 I_1 I_2}{4\pi R^2}\right)\left(\overline{d\ell_1} \cdot \overline{d\ell_2}\right) \hat{\mathbf{a}}_{21}$$

Therefore, the force on filament **1** has a component in the direction $\overline{d\ell_2}$ (i.e., in the direction of current filament **2**), **and** a component in the direction $-\hat{a}_{21}$.



So, let's consider several **examples**:

Example 1: Filament **2** points **toward** filament **1**



Therefore, since $\overline{d\ell_2} = \left|\overline{d\ell_2}\right|\hat{a}_{21}$:

$$\begin{aligned}\left(\overline{d\ell_1}\cdot\hat{a}_{21}\right)\overline{d\ell_2} &= \left(\overline{d\ell_1}\cdot\hat{a}_{21}\right)\left|\overline{d\ell_2}\right|\hat{a}_{21}\\ &= \left(\overline{d\ell_1}\cdot\left|\overline{d\ell_2}\right|\hat{a}_{21}\right)\hat{a}_{21}\\ &= \left(\overline{d\ell_1}\cdot\overline{d\ell_2}\right)\hat{a}_{21}\end{aligned}$$

and therefore:

$$d\mathbf{F}_1^a = -\left(\frac{\mu_0 I_1 I_2}{4\pi R^2}\right)\left(\overline{d\ell_1}\cdot \hat{a}_{21}\right)\overline{d\ell_2}$$

$$= -\left(\frac{\mu_0 I_1 I_2}{4\pi R^2}\right)\left(\overline{d\ell_1}\cdot \overline{d\ell_2}\right)\hat{a}_{21}$$

$$= -d\mathbf{F}_1^b$$

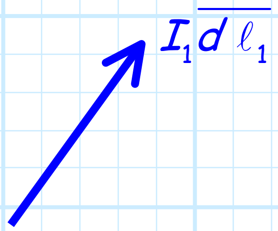
And thus:

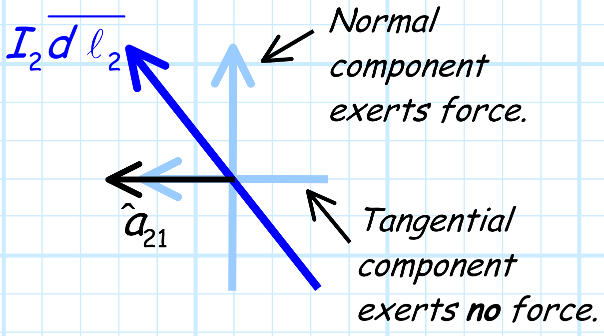
$$d\mathbf{F}_1 = d\mathbf{F}_1^a + d\mathbf{F}_1^b$$

$$= -d\mathbf{F}_1^b + d\mathbf{F}_1^b = 0$$

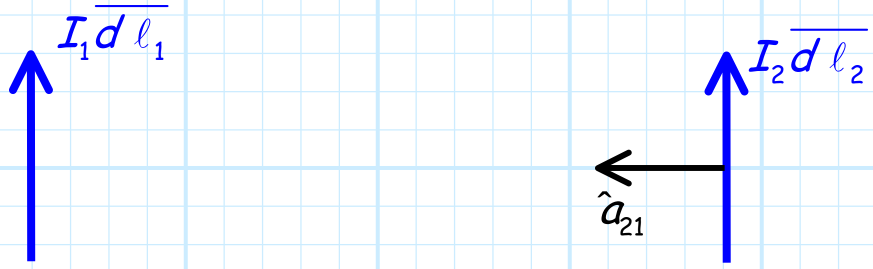
In other words, if filament 2 **points** at filament 1, then the force on filament 1 is **zero**, **regardless** of the orientation of filament 1.

Another way of saying this is that **only** the component of $I_2\overline{d\ell_2}$ that is **orthogonal** to $\hat{a}_{21}$ can exert of force on filament 1.





Example 2: Filament 1 is **parallel** to filament 2



Therefore,

$$\overline{d\ell_1} \cdot \hat{a}_{21} = 0$$

so:

$$d\mathbf{F}_1^a = \left(\frac{\mu_0 I_1 I_2}{4\pi R^2}\right)\left(\overline{d\ell_1} \cdot \hat{a}_{21}\right)\ \overline{d\ell_2} = 0$$

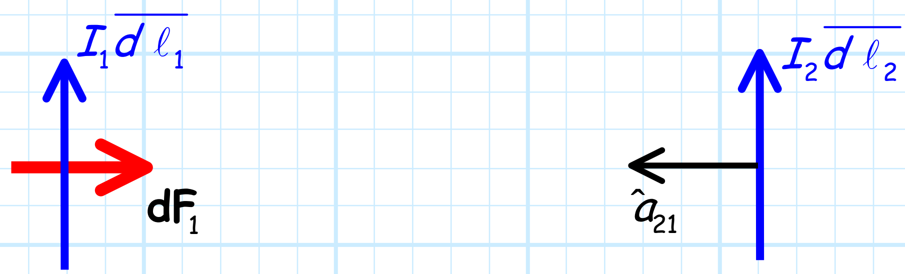
But,

$$\overline{d\ell_1} \cdot \overline{d\ell_2} \neq 0$$

therefore:

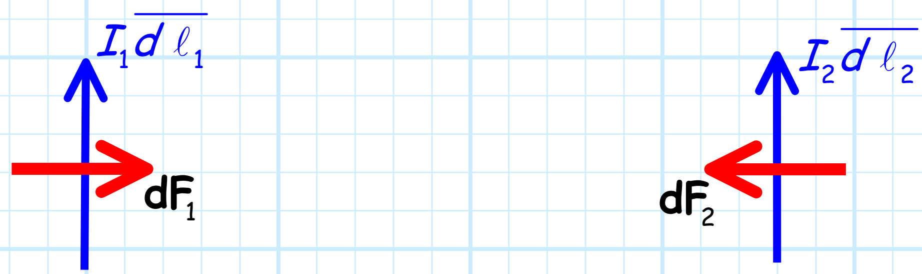
$$d\mathbf{F}_1^b = -\left(\frac{\mu_0 I_1 I_2}{4\pi R^2}\right)\left(\overline{d\ell_1} \cdot \overline{d\ell_2}\right)\ \hat{a}_{21} \neq 0$$

Thus, $d\mathbf{F}_1 = d\mathbf{F}_1^b$, applying a force in the direction $-\hat{a}_{21}$!

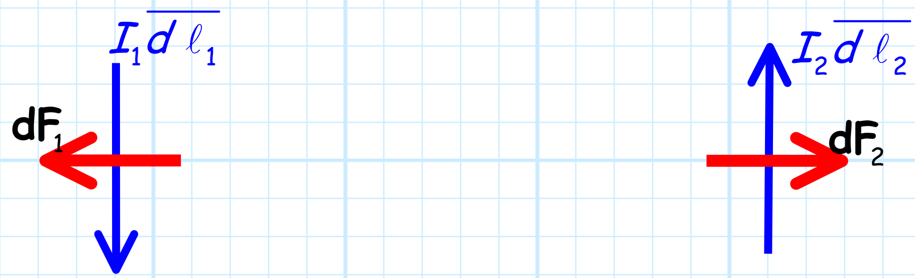


Filament 1 is **attracted** to filament 2 !

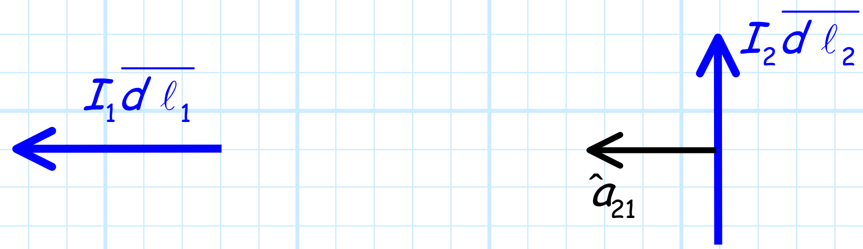
For the same reasons, filament 2 is attracted to filament 1:



But, we find that the two filaments **repel** if they point in **opposite** directions:



Example 3: Filament 1 is **parallel** to $\hat{a}_{21}$ and **orthogonal** to filament 2.



Therefore,

$$\overline{d\ell_1} \cdot \overline{d\ell_2} = 0$$

so:

$$d\mathbf{F}_1^b = -\left(\frac{\mu_0 I_1 I_2}{4\pi R^2}\right)\left(\overline{d\ell_1} \cdot \overline{d\ell_2}\right)\ \hat{a}_{21} = 0$$

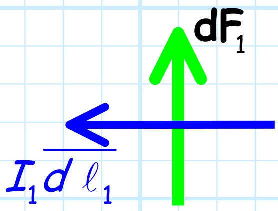
But,

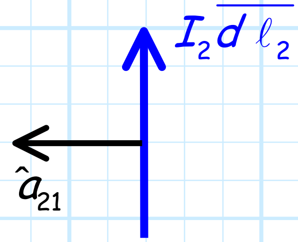
$$\overline{d\ell_1} \cdot \hat{a}_{21} \neq 0$$

thus:

$$d\mathbf{F}_1^a = \left(\frac{\mu_0 I_1 I_2}{4\pi R^2}\right)\left(\overline{d\ell_1} \cdot \hat{a}_{21}\right)\ \overline{d\ell_2} \neq 0$$

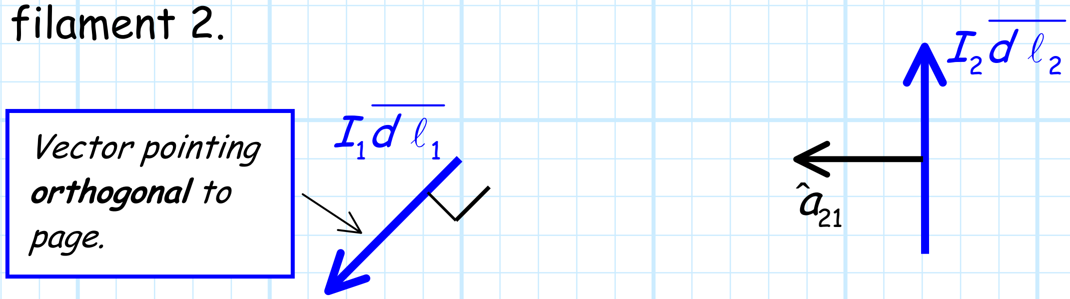
Therefore, $d\mathbf{F}_1 = d\mathbf{F}_1^a$, applying a force in the direction $\overline{d\ell_2}$:





Note however, the force on filament **2** is **zero** !

Example 4: Filament 1 is **orthogonal** to $\hat{a}_{21}$ and **orthogonal** to filament 2.



In this case, we find:

$$\overline{d\ell_1} \cdot \hat{a}_{21} = 0$$

so:

$$d\mathbf{F}_1^a = \left(\frac{\mu_0 I_1 I_2}{4\pi R^2}\right)\left(\overline{d\ell_1} \cdot \hat{a}_{21}\right) \ \overline{d\ell_2} = 0$$

Likewise,

$$\overline{d\ell_1} \cdot \overline{d\ell_2} = 0$$

thus:

$$d\mathbf{F}_1^b = -\left(\frac{\mu_0 I_1 I_2}{4\pi R^2}\right)\left(\overline{d\ell_1} \cdot \overline{d\ell_2}\right) \ \hat{a}_{21} = 0$$

Therefore, the total force on filament 1 is **zero**:

$$d\mathbf{F}_1 = d\mathbf{F}_1^a + d\mathbf{F}_1^b = 0$$

For the same reasons, we find that the force on filament 2 due to filament 1 is **also zero** (i.e., $d\mathbf{F}_2 = 0$).